[a] $(2tx - 2x - 4t^2\sqrt{x})dt + t\,dx = 0$ **FINAL ANSWER**: $x = (2t + Cte^{-t})^2$

TOTAL

10

$$2tx - 2x - 4t^2\sqrt{x} + t\tfrac{dx}{dt} = 0$$

$$\boxed{\tfrac{dx}{dt} + (2 - \tfrac{2}{t})x = 4t\sqrt{x}}$$ ⬅ BERNOULLI $n = \frac{1}{2}$

Let $\boxed{v = x^{1-\frac{1}{2}} = x^{\frac{1}{2}}}$

So $\frac{dv}{dt} = \frac{1}{2}x^{-\frac{1}{2}}\frac{dx}{dt}$ and $\frac{dx}{dt} = 2x^{\frac{1}{2}}\frac{dv}{dt}$

$$\boxed{2x^{\frac{1}{2}}\tfrac{dv}{dt} + (2 - \tfrac{2}{t})x = 4t\sqrt{x}}$$

$$\tfrac{dv}{dt} + (1 - \tfrac{1}{t})x^{\frac{1}{2}} = 2t$$

$$\boxed{\tfrac{dv}{dt} + (1 - \tfrac{1}{t})v = 2t}$$ ⬅ LINEAR

$$\boxed{\mu = e^{\int(1-\frac{1}{t})dt} = e^{t - \ln|t|} = t^{-1}e^{t}}$$

$$\boxed{t^{-1}e^{t}\tfrac{dv}{dt} + (t^{-1} - t^{-2})e^{t}v = 2e^{t}}$$

$\boxed{\text{CHECK: } \tfrac{d}{dt}t^{-1}e^{t} = -t^{-2}e^{t} + t^{-1}e^{t} \checkmark}$

$$\boxed{t^{-1}e^{t}v = \int 2e^{t}dt = 2e^{t} + C}$$

$\boxed{v = 2t + Cte^{-t}}$ $\frac{1}{2}$

$\boxed{x^{\frac{1}{2}} = 2t + Cte^{-t}}$ $\frac{1}{2}$

$\boxed{x = (2t + Cte^{-t})^2}$

ALL ITEMS 1 POINT ON ALL QUESTIONS UNLESS OTHERWISE INDICATED

[b] $\theta(\theta - r)r' - \theta r = r^2$ **FINAL ANSWER:** $r\theta e^{\frac{\theta}{r}} = C$

TOTAL $8\frac{1}{2}$

$$\theta(\theta - r)\tfrac{dr}{d\theta} - \theta r - r^2 = 0$$

$$(\theta^2 - r\theta)dr + (-\theta r - r^2)d\theta = 0$$

$$(t\theta)^2 - (tr)(t\theta) = t^2(\theta^2 - r\theta) \text{ and } -(t\theta)(tr) - (tr)^2 = t^2(-\theta r - r^2)$$ ← HOMOGENEOUS

Let $r = v\theta$

$$(\theta^2 - v\theta^2)(vd\theta + \theta dv) + (-v\theta^2 - v^2\theta^2)d\theta = 0$$

$$(1 - v)(vd\theta + \theta dv) + (-v - v^2)d\theta = 0$$

$$(-v - v^2 + v - v^2)d\theta + (1 - v)\theta dv = 0$$

$$-2v^2 d\theta + (1 - v)\theta dv = 0$$

$$2v^2 d\theta = (1 - v)\theta dv$$

$$\int \tfrac{2}{\theta} d\theta = \int (v^{-2} - v^{-1})dv$$ ← SEPARABLE

$$2\ln|\theta| + C = -v^{-1} - \ln|v|$$

$$2\ln|\theta| + C = -\tfrac{\theta}{r} - \ln\left|\tfrac{r}{\theta}\right|$$ $\frac{1}{2}$

$$2\ln|\theta| + C = -\tfrac{\theta}{r} - \ln|r| + \ln|\theta|$$

$$\ln|\theta| + C = -\tfrac{\theta}{r} - \ln|r|$$

$$C\theta = e^{-\frac{\theta}{r}} r^{-1}$$

$$r\theta e^{\frac{\theta}{r}} = C$$

ALTERNATE SOLUTION

GRADE USING ONLY ONE SOLUTION

[b] $\theta(\theta - r)r' - \theta r = r^2$ **FINAL ANSWER:** $r\theta e^{\frac{\theta}{r}} = C$

$$\theta(\theta - r)\tfrac{dr}{d\theta} - \theta r - r^2 = 0$$

$$(\theta^2 - r\theta)dr + (-\theta r - r^2)d\theta = 0$$

$$(t\theta)^2 - (tr)(t\theta) = t^2(\theta^2 - r\theta) \text{ and } -(t\theta)(tr) - (tr)^2 = t^2(-\theta r - r^2)$$ ← HOMOGENEOUS

Let $\theta = vr$

$$(v^2r^2 - vr^2)dr + (-vr^2 - r^2)(vdr + rdv) = 0$$

$$(v^2 - v)dr + (-v - 1)(vdr + rdv) = 0$$

$$(v^2 - v - v^2 - v)dr + (-v - 1)rdv = 0$$

$$-2vdr + (-v - 1)rdv = 0$$

$$-2vdr = (v + 1)rdv$$

$$\int -\tfrac{2}{r} dr = \int (1 + v^{-1})dv$$ ← SEPARABLE

$$-2\ln|r| + C = v + \ln|v|$$

$$-2\ln|r| + C = \tfrac{\theta}{r} + \ln\left|\tfrac{\theta}{r}\right|$$ $\frac{1}{2}$

$$-2\ln|r| + C = \tfrac{\theta}{r} + \ln|\theta| - \ln|r|$$

$$C = \tfrac{\theta}{r} + \ln|\theta| + \ln|r|$$

$$r\theta e^{\frac{\theta}{r}} = C$$

[c] $\dfrac{dy}{dx} = \dfrac{\sin x \cos y}{1 + \cos x \sin y}$

FINAL ANSWER: $\cos x + \sin y = C \cos y$

TOTAL 12

$$-\sin x \cos y\,dx + (1 + \cos x \sin y)dy = 0$$

$$M = -\sin x \cos y \quad \Rightarrow \quad M_y = \sin x \sin y \quad \tfrac{1}{2}$$

$$N = 1 + \cos x \sin y \quad \Rightarrow \quad N_x = -\sin x \sin y \quad \tfrac{1}{2}$$

$$\frac{N_x - M_y}{M} = \frac{-2\sin x \sin y}{-\sin x \cos y} = \frac{2\sin y}{\cos y}$$ ⬅ FUNCTION OF ONLY y

$$\mu = e^{\int \frac{2\sin y}{\cos y} dy} = e^{-2\ln|\cos y|} = \sec^2 y$$

$$-\sin x \sec y\,dx + (\sec^2 y + \cos x \sec y \tan y)dy = 0$$

$$M = -\sin x \sec y \quad \Rightarrow \quad M_y = -\sin x \sec y \tan y$$

$$N = \sec^2 y + \cos x \sec y \tan y \quad \Rightarrow \quad N_x = -\sin x \sec y \tan y \checkmark$$ ⬅ EXACT

$$f = \int -\sin x \sec y\,dx \quad \Rightarrow \quad f = \cos x \sec y + C(y)$$

$$f_y = \cos x \sec y \tan y + C'(y) = \sec^2 y + \cos x \sec y \tan y \quad \Rightarrow \quad C'(y) = \sec^2 y \quad \Rightarrow \quad C(y) = \tan y$$

$\tfrac{1}{2}$ $\tfrac{1}{2}$

$$f = \cos x \sec y + \tan y = C$$

$$\cos x + \sin y = C \cos y$$